Book Review of: “Hidden Dynamics: The Mathematics of Switches, Decisions and Other Discontinuous Behaviour” (by Mike Jeffrey)

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The first key question I always try to find out by reading a book review is, how the work under review fits into the existing monograph and textbook landscape in the area? Are there other comparable alternatives? Is the new book particularly timely? Do we urgently need a book on the subject? So let me start with these questions in mind.

Non-smooth differential equations have a long history. Even the modern dynamics theory of the subject reaches back at least to the middle of the 20th century. Yet, there are still relatively few available books organizing the progress in the subject. The first landmark monograph covering substantial efforts of the Russian mathematical and engineering school is the book by Filippov [3] from 1988 (respectively from 1985 in its original Russian version). Later on one finds Brogliato’s work with a focus on mechanics and control [1] as well as elegant shorter accounts by Kunze [7] and by Leine and Nijmeijer [9]. The last two books with a strong nonlinear dynamical systems focus on non-smooth system are the monographs by di Bernardo, Budd, Champneys and Kowalczyk [2] from 2008, and the book by Simpson from 2010 [10]. Based on this bird’s eye view, it is evident to conclude that we not only need another book summarizing the rapid recent progress, actually such a book is long overdue! In fact, this was a key initial reason, why I was particularly interested in Jeffrey’s contribution to non-smooth systems [5].

The book has fourteen chapters. The first two are introductory and provide a general perspective as well as as a concrete setting for planar systems. In the planar setup the geometric view is particularly appealing. Planar non-smooth ordinary differential equations (ODEs) serve as zeroth-order benchmark problems for the entire subject. Instead of a differentiable, or even continuous, vector field, one studies systems with discontinuities, which are assumed to occur on submanifolds of codimension one or higher. For planar piecewise smooth ODEs, one typically focuses on a single curve separating two regions in which the vector field is smooth. On the curve itself there is no a-priori given definition of the flow. Without additional knowledge we have to define the dynamics on the curve, where one switches between vector fields. The term “hidden dynamics” can be interpreted as capturing the need use the given vector fields and potentially even additional hidden terms, to uncover dynamics near a switching curve. Of course, a similar philosophy is then applied to switching for higher-dimensional surfaces/manifolds/varieties. These geometric objects are also called switching layers. The theme of focusing on the effects induced by a switching layer gets exploited throughout the last twelve chapters of the book. The stage is set in Chapters 3-5 with definitions of non-smooth flows, trajectories, switching multipliers, sliding, and related concepts. Chapters 6-8 cover somewhat classical material, or at least relate to classical constructions ranging from singularity theory to (linear) stability and bifurcations. In this context, two key differences for non-smooth ODEs are that the vector field can be tangent to a switching layer and that equilibria may lie on the switching layer. These issues induce already considerable difficulty for piecewise linear ODEs. Chapters 9-13 attack these issues for nonlinear problems. These chapters form the theoretical cornerstone contribution of the book by Jeffrey. The presentation has a focus on bifurcation phenomena, breaking of determinancy via multiple possible trajectory extensions near switching, desingularization, and finally culminating in a detailed analysis of the two-fold singularity. Via desingularization of the switching layer, one actually observes that desingularized smooth systems display phenomena very close to, or even exactly studied already, in the context of multiple time scale dynamics [6]. The dynamics near the switching layer becomes slow, while the external vector fields are governed by fast motion. Finally, in Chapter 14, Jeffrey discusses several modern applications of the theory.

Instead of delving deeper into the material, let me continue to a general view on the book. The first thing one notices, when starting to read is that the style is quite different from a "classical" mathematics monograph. It is neither written in a pure mathematics theorem-proof format, nor in a purely example-application driven format. Instead it seems to me that Jeffrey's intention is to directly talk applied mathematics to the reader and to communicate his views and excitement for non-smooth geometric dynamics. In my personal opinion, we need more bravery like this in book writing. Trying a different nonstandard style in such a major undertaking like a book takes courage and should be applauded. Of course, whether the creative approach has worked for this book can probably only be decided in one, or potentially even more, decades by the current and future readers.

So what is the target audience? I think the book is very readable, even with a relatively low-level background. Probably the minimum would be a first course in nonlinear dynamical systems [11] although a second course certainly will be helpful as well [4], potentially with a focus on bifurcation theory [8] or multiscale systems [6] as these subjects link particularly tightly with the viewpoint of Jeffrey. Therefore, the potential audience is actually very broad including all research-oriented applied mathematicians with an interest in non-smooth systems. In terms of using it as a textbook, one is charged to extract the essential messages out of Jeffrey's book as well as from the the other sources mentioned above [2, 7, 10, 9, 3, 1] to really provide a comprehensive short mathematical course on the topic. Yet, Jeffrey's book seems ideally suited for a master-level seminar course, where students present the book chapter-by-chapter to each other.

The last, potentially most debatable, question I would like to raise here concerns the general paradigm of non-smooth systems. Particularly due to their high practical use for mechanical systems in the context of impact and friction, their practical usefulness is established. However, to find the "best" mathematical modelling viewpoint is still somewhat under discussion. Jeffrey points out these issues (rightly so!) already in the beginning of his book and the discussion recurs throughout at various points. For example, one may argue that upon mathematically zooming in or using a finer model, we may smooth a system, so why bother with the non-smooth view? Here I fully agree with Jeffrey's approach that the non-smooth system is a very useful singular limit, and that smoothing should be brought in as a technical tool. The second key point made often in the context of non-smooth models is to observe the evident non-uniqueness induced by a switching layer, and then to analyze the resulting non-deterministic dynamics. So far, so good. Yet, this approach really involves in a vast majority only classical techniques from deterministic analysis, almost pathologically avoiding probabilistic methods, with very few recent exceptions. Jeffrey's book is not one of these exceptions. He focuses on the "deterministic analysis of non-determinism". Personally, I think the stochastic approach holds a lot of promise to better understand apparent non-smooth loss of determinism. Not only does "stochastic analysis of non-determinism" sound natural to me but it may bring fundamental new insights in the next decades (and potentially lead to another useful book!).

In summary, I can definitely recommend Jeffrey's book for anyone looking to learn more about non-smooth dynamical systems. Jeffrey lays out his view on hidden dynamics and thereby also sets out a potential future agenda. Therefore, the book is one little puzzle piece, which is hopefully going motivate mathematicians of various interests to re-fine and extend methods in the vibrant area of applied dynamical systems.

References

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